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Topological Indices of Complete Graph with a Single rooted Vertex

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Abstract- A topological index of a chemical compound characterizes the compound and obeys a particular rule. In this paper, we find the Harmonic Index and the custom defined Redefined Zagreb Indices of Complete Graph with a single rooted vertex.

Index Terms- Complete graph, Harmonic Index, Redefined Zagreb Indices, Single rooted vertex.

1 Introduction

In this paper, all graphs used are simple and undirected for the reason that there would not be any multiple edges or loops in the intended and resulting structure. Let G be a simple graph, the vertex-set and edge-set of which are represented by V(G) and E(G) respectively. If u and v are two vertices of G then $d_G(u; v)$ denotes the length of the shortest path connecting u and v .

An (n; k) banana tree, as defined by [1], is a graph obtained by connecting one leaf of each of n copies of a k star graph with a single root vertex that is distinct from all the stars. Motivated by the graphs of banana tree, the work extends the same concept of single rooted vertex attached to a complete graph. So, it is denoted by $BG_{m;n}$ where m is the number of complete graphs attached and n is the total number of vertices.

A complete graph is a graph in which each pair of graph vertices is connected by an edge. The complete graph with n graph vertices is denoted K_n and has n(n 1)/2 undirected edges.

The Harmonic Index [7], [4] of a graph G is defined by

$$H(G) = \sum \frac{2}{d_u + d_v}$$

Where (u,v) is an element of E(G).

Motivated by the definition of Zagreb Indices [2, 3, 5, 6], an attempt to define new indices are made. The Redefined First Zagreb Index of a graph G is defined by

$$ReZG_1(G) = \sum \frac{d_u + d_v}{d_u . d_v}$$

Where (u,v) is an element of E(G).

The Redefined Second Zagreb Index of a graph G is defined by

$$ReZG_2(G) = \sum \frac{d_u . d_v}{d_u + d_v}$$

The Redefined Third Zagreb Index of a graph G is defined by

$$ReZG_3(G) = \sum [\, d_u + d_v]. [d_u . d_v]$$

2. Harmonic Index, Redefined Zagreb Indices of Complete Graph with a Single Rooted Vertex

Theorem 2.1 The Harmonic Index, and the Redefined Zagreb Indices of a Complete Graph with a single rooted vertex $BG_{(m,3)}$ where n≥7, where n is the total number of vertices and m is the number of complete graphs attached to a single rooted vertex are

$$H(BG_{m,3}) = \frac{13(n-1)}{30} + \frac{2m}{2+m}$$

$$ReZG_1(BG_{m,3}) = \frac{8n-8}{9} + \frac{3+m}{3}$$

$$ReZG_2(BG_{m,3}) = \frac{17n-17}{15} + \frac{3m^2}{3+m}$$

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$$ReZG_3(BG_{m,3}) = \frac{76n-76}{3} + 3m^2(3+m)$$

Proof: There are (3x+4) number of vertices, where x takes the values 1,2,3,.. and the number of edges are 4(n-1)/3. A complete graph with a single rooted vertex $(BG_{m,3})$ will have edges whose end vertices are (2,2),(2,3) and (3,m). Considering the number of edges of all the types and calculating the above indices will result in the following.

$$H(BG_{m,3}) = \frac{13(n-1)}{30} + \frac{2m}{2+m}$$

$$ReZG_1(BG_{m,3}) = \frac{8n-8}{9} + \frac{3+m}{3}$$

$$ReZG_2(BG_{m,3}) = \frac{17n-17}{15} + \frac{3m^2}{3+m}$$

$$ReZG_3(BG_{m,3}) = \frac{76n-76}{3} + 3m^2(3+m)$$

Theorem 2.2: The Harmonic Index, and the Redefined Zagreb Indices of a Complete Graph with a single rooted vertex $BG_{(m,4)}$ where n≥9, where n is the total number of vertices and m is the number of complete graphs attached to a single rooted vertex are

$$H(BG_{m,4}) = \frac{15(n-1)}{30} + \frac{2m}{4+m}$$

$$ReZG_1(BG_{m,4}) = \frac{15n-15}{16} + \frac{4+m}{4}$$

$$ReZG_2(BG_{m,4}) = \frac{135n-135}{56} + \frac{4m^2}{4+m}$$

$$ReZG_3(BG_{m,4}) = \frac{207n-207}{2} + 4m^2(4+m)$$

Proof: There are (4x+5) number of vertices, where x takes the values 1,2,3,.. and the number of edges are (n-1). A complete graph with a single rooted vertex $(BG_{m,4})$ will have edges whose end vertices are (3,3),(3,4) and (4,m). Considering the number of edges of all the types and calculating the above indices will result in the following.

$$H(BG_{m,4}) = \frac{15(n-1)}{30} + \frac{2m}{4+m}$$

$$ReZG_1(BG_{m,4}) = \frac{15n-15}{16} + \frac{4+m}{4}$$

$$ReZG_2(BG_{m,4}) = \frac{135n-135}{56} + \frac{4m^2}{4+m}$$

$$ReZG_3(BG_{m,4}) = \frac{207n-207}{2} + 4m^2(4+m)$$

Theorem 2.3: The Harmonic Index, and the Redefined Zagreb Indices of a Complete Graph with a single rooted vertex $BG_{(m,5)}$ where n≥11, where n is the total number of vertices and m is the number of complete graphs attached to a single rooted vertex are

$$H(BG_{m,5}) = \frac{23(n-1)}{75} + \frac{2m}{2+m}$$

$$ReZG_1(BG_{m,5}) = \frac{24n-24}{25} + \frac{5+m}{5}$$

$$ReZG_2(BG_{m,4}) = \frac{188n-188}{45} + \frac{5m^2}{5+m}$$

$$ReZG_3(BG_{m,4}) = \frac{1488(n-1)}{2} + 5m^2(5+m)$$

Proof: There are (5x+6) number of vertices, where x takes the values 1,2,3,.. and the number of edges are (n-1). A complete graph with a single rooted vertex $(BG_{m,5})$ will have edges whose end vertices are (4,4),(4,5) and (5,m). Considering the number of edges of all the types and calculating the above indices will result in the following.

$$H(BG_{m,5}) = \frac{23(n-1)}{75} + \frac{2m}{2+m}$$

$$ReZG_1(BG_{m,5}) = \frac{24n-24}{25} + \frac{5+m}{5}$$

$$ReZG_2(BG_{m,5}) = \frac{188n-188}{45} + \frac{5m^2}{5+m}$$

$$ReZG_3(BG_{m,5}) = \frac{1488(n-1)}{2} + 5m^2(5+m)$$

Theorem 2.4: The Harmonic Index, and the Redefined Zagreb Indices of a Complete Graph with a single rooted vertex $BG_{(m,6)}$ where n≥13, where n is the total number of vertices and m is the number of complete graphs attached to a single rooted vertex are

$$H(BG_{m,6}) = \frac{85(n-1)}{198} + \frac{2m}{6+m}$$

$$ReZG_1(BG_{m,6}) = \frac{31n-31}{36} + \frac{6+m}{6}$$

$$ReZG_2(BG_{m,6}) = \frac{80n-80}{11} + \frac{6m^2}{6+m}$$

$$ReZG_3(BG_{m,6}) = (995n-995) + 6m^2(6+m)$$

Proof: There are (6x+7) number of vertices, where x takes the values 1,2,3,.. and the number of edges are (n-1). A complete graph with a single rooted vertex ($BG_{m,6}$) will have edges whose end vertices are (5,5),(5,6) and (6,m). Considering the number of edges of all the types and calculating the above indices will result in the following.

$$H(BG_{m,6}) = \frac{85(n-1)}{198} + \frac{2m}{6+m}$$

$$ReZG_1(BG_{m,6}) = \frac{31n-31}{36} + \frac{6+m}{6}$$

$$ReZG_2(BG_{m,6}) = \frac{80n-80}{11} + \frac{6m^2}{6+m}$$

$$ReZG_3(BG_{m,6}) = (995n-995) + 6m^2(6+m)$$

Theorem 2.5: : The Harmonic Index, and the Redefined Zagreb Indices of a Complete Graph with a single rooted vertex $BG_{(m,n)}$ where n≥9, where n is the total number of vertices and m is the number of complete graphs attached to a single rooted vertex are

$$H(BG_{m,n}) = \frac{(2n^2-n-2)(n-1)}{2n(2n-1)} + \frac{2m}{(m+n)}$$

$$ReZG_1(BG_{m,n}) = \frac{(n-1)^2(n+1)}{n^2} + \frac{n+m}{n}$$

$$ReZG_2(BG_{m,n}) = \frac{(n-1)^3(2n^2-n+2)}{4n(2n-1)} + \frac{m^2}{m+n}$$

$$ReZG_3(BG_{m,n}) = \frac{(n-1)^3(n^3-2n^2+4n-2)}{n} + nm^2(m+n)$$

Proof: In a complete graph with a single rooted vertex ($BG_{m,n}$), there are nx+(n+1) number of vertices, where x takes the values 1,2,3,.. and the number of edges are (n^2-n+2)/2. A complete graph with a single rooted vertex ($BG_{m,n}$) will have edges whose end vertices are (n-1,n-1), (n-1,n) and (n,m). Considering the number of edges of all the types and calculating the above indices will result in the following.

$$H(BG_{m,n}) = \frac{(2n^2-n-2)(n-1)}{2n(2n-1)} + \frac{2m}{(m+n)}$$

$$ReZG_1(BG_{m,n}) = \frac{(n-1)^2(n+1)}{n^2} + \frac{n+m}{n}$$

$$ReZG_2(BG_{m,n}) = \frac{(n-1)^3(2n^2-n+2)}{4n(2n-1)} + \frac{m^2}{m+n}$$

$$ReZG_3(BG_{m,n}) = \frac{(n-1)^3(n^3-2n^2+4n-2)}{n} + nm^2(m+n)$$

3.References

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